

The authors examine combined heat and mass transfer in conditions of natural convection on a horizontal slab at large Rayleigh number Ra , in the case where variation of density due to variation of concentration near the surface is negligibly small in comparison with the change of density due to the drop in temperatures. This case is met with in practice, for example, during intense heating or cooling of weak solutions or gas mixtures depleted with respect to a surface active component, and interacting on the surface.

1. Analysis of the system of differential equations describing heat and mass transfer under steady-state natural convection shows that the presence of mass transfer in the conditions examined does not influence the heat transfer and the hydrodynamics of the process. Therefore, in this case the intensity of heat transfer for $Ra > 2 \cdot 10^7$, as in the case of ordinary thermal convection, can be calculated from the relation [1]

$$Nu = 0.18(GrPr)^{1/3}, \quad (1.1)$$

where the physical properties are chosen at the average liquid temperature. The relation for the mass transfer intensity can be obtained from the Leon'tev—Kirdyashkin theory [2, 3] for single-phase cellular thermal convection on a horizontal slab. According to [2], in the case of thermal convection at Rayleigh numbers in excess of a specific critical value Ra_* , a cellular layer is formed on the horizontal heater surface, and the intensity of liquid motion in it governs both the heat transfer and also the mass transfer in our case.

Figure 1, taken from [3], shows a postulated scheme for the circulation of liquid in the case of cellular convection in a large volume at large Rayleigh numbers. The theory of [2, 3] is based on the postulate of constant vorticity of the liquid in the cells. Here it is assumed that at the heater surface the horizontal component of the velocity of the external flow varies along the surface according to the sinusoidal law

$$v_0 = v_m \sin \pi \bar{x}, \quad (1.2)$$

where v_m is the maximum liquid velocity at the heater surface; $\bar{x} = x/l$; x is the longitudinal coordinate; and l is the linear dimension of the cell. From a combined examination of Eq. (1.2) and the equations for the hydrodynamic and thermal boundary layers arising at the heater surface due to the swirling flow we can obtain relations for the maximum velocity, the size of the cell, and the heat transfer intensity [2, 3]:

$$v_m = 0.85(\beta_r g l \Delta T / Pr^{0.6})^{1/2}; \quad (1.3)$$

$$l = (av Ra_* / (\Delta T \beta_r g))^{1/3}; \quad (1.4)$$

$$Nu = 0.49 Ra_*^{-1/12} (Gr Pr)^{1/3}. \quad (1.5)$$

The characteristic length here is the linear cell dimension. When we substitute the critical value $Ra_* = 3100$ [3] we obtain the value 0.25 for the constant coefficient of $(GrPr)^{1/3}$ in Eq. (1.5). It was shown in [3] that when one allows for the finite size of the thermal boundary layer thickness one obtains a coefficient of 0.22 in Eq. (1.5). The fact that the heat transfer is independent of the characteristic length allows us to compare Eqs. (1.1) and (1.5) in which this length is chosen to be different, generally speaking. The comparison shows that, although the numerical coefficient of 0.22 is rather large compared with the value in the experimental formula, nevertheless the relation between the Nu number and the governing parameters in Eq. (1.5) is quite correct, in spite of the approximate nature of the theory.

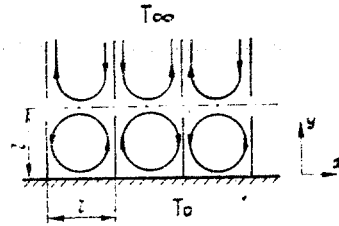


Fig. 1

To determine the mass transfer one must take into account, analogously as was done in the analysis of heat transfer [2, 3], the additional equation of the diffusion boundary layer with $\Delta c_0 = \text{const}$ and give the mass transfer law for the boundary layer:

$$d\text{Re}_c^{**}/d\bar{x} = \text{Re}_l \text{St}_c; \quad (1.6)$$

$$\text{St}_c = 0.22/(\text{Re}_c^{**} \text{Sc}^m), \quad (1.7)$$

where $\text{Re}_c^{**} = v_0 \delta_c^{**}/\nu$; $\text{Re}_l = v_0 l/\nu$; $\text{St}_c = \beta/v_0$; and δ_c^{**} is the material loss thickness.

From Eqs. (1.6) and (1.7), allowing for Eq. (1.2) it follows that

$$\text{Re}_c^{**} = \left[\frac{0.44 v_0 l}{\pi \nu \text{Sc}^m} (1 - \cos \pi \bar{x}) \right]^{1/2}, \quad \beta = 0.59 \text{Sc}^{(1-m/2)} D \sqrt{\frac{v_0}{l \nu} \frac{\sin \pi \bar{x}}{1 - \cos \pi \bar{x}}}.$$

The average value of the mass transfer coefficient is

$$\bar{\beta} = 0.53 D \text{Sc}^{(1-m/2)} \sqrt{v_m/l \nu}. \quad (1.8)$$

Substituting v_m and l from Eqs. (1.3) and (1.4) into Eq. (1.8), and reducing the equation to dimensionless form, we obtain

$$\text{Sh} = 0.49 \text{Ra}_*^{-1/12} \text{Pr}^{(1.2-m)/3} \text{Le}^{(4-3m)/6} (\text{GrSc})^{1/3}. \quad (1.9)$$

For gases, where the numbers $\text{Pr} \sim \text{Sc} \sim 1$, the exponent in Eq. (1.7), as is true for the corresponding relation in heat transfer [3], is 1.2. Substituting $m = 1.2$ into Eq. (1.9) we have

$$\text{Sh} = 0.49 \text{Ra}_*^{-1/12} \text{Le}^{1/15} (\text{GrSc})^{1/3}. \quad (1.10)$$

The expressions for the constants in the theoretical formulas of Eqs. (1.5) and (1.10) have the same accuracy, and therefore the errors in determining these coefficients must be the same. Taking this into account, one should choose the constant in Eq. (1.10) to be the same as in the experimental formula for the heat transfer, Eq. (1.1). Then Eq. (1.10) takes the form

$$\text{Sh} = 0.18 \text{Le}^{1/15} (\text{GrSc})^{1/3}. \quad (1.11)$$

Liquids, as is known, have typically large Sc numbers. According to [4], the exponent m in Eq. (1.7) should increase with increase of Sc , and reach the value $m = 4/3$ in the limit as $\text{Sc} \rightarrow \infty$. Therefore, for liquids characterized by numbers $\text{Sc} > 10$, the value of m must be chosen in the range $1.2 < m < 4/3$. Taking $m = (1.2 + 4/3)/2$, in accordance with Eq. (1.9) we can obtain

$$\text{Sh} = 0.18 \text{Le}^{1/30} \text{Pr}^{-1/24} (\text{GrSc})^{1/3}. \quad (1.12)$$

For the limiting case $\text{Sc} \rightarrow \infty$ ($m = 4/3$) we have

$$\text{Sh} = 0.18 \text{Pr}^{-1/12} (\text{GrSc})^{1/3}. \quad (1.13)$$

Thus, the intensity of mass transfer, in natural cellular thermal convection on a horizontal slab, in the case of a weak variation of density of the medium due to the presence of a concentration gradient at the heater surface, is determined basically by the single dimensionless parameter GrSc . For $\text{Sc} \sim 1$ there is a weak dependence of the mass transfer intensity on Le . With increase of Sc the dependence of mass transfer intensity on Le becomes weaker, and a certain dependence of Nu on Pr appears. In sum the variation of the mass transfer intensity with variation of Pr and Le over their entire possible range in practice does not exceed 10-30%.

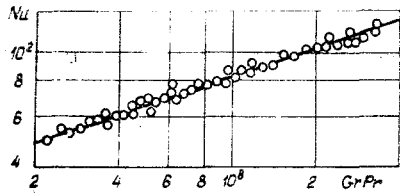


Fig. 2

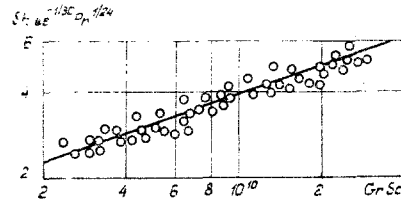
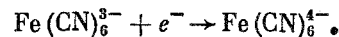


Fig. 3

2. The mass transfer process was investigated experimentally using the electrodiffusion technique. As the working liquid we used a $0.005\text{NK}_3[\text{Fe}(\text{CN})_6] + 0.01\text{NK}_4[\text{Fe}(\text{CN})_6] + 0.5\text{NNaOH}$, aqueous solution whose thermophysical properties as a function of temperature were determined in a manner analogous to that of [5, 6]. The heat and mass transfer intensity was investigated on the nickel-plated face of a cylindrical copper heater of diameter 50 mm positioned so that the heat transfer surface and the base of the chamber lay in one plane. As the measuring electrode (cathode) we used the entire heater surface. When current passed through the solution there was a unique thermochemical reaction of first order on the surface of the electrodiffusion sensor



The measured electric current in the circuit of the measuring electrode is a measure of the mass transfer intensity on the surface of the electrodiffusion sensor. The mass transfer coefficient in the limiting diffusion regime ($c_0 = 0$) is calculated from the formula

$$\beta = I_{\text{lim}} / AFc_\infty,$$

where I_{lim} is the limiting diffusion current; A is the Faraday number; F is the electrode area; c_∞ and c_0 are the concentration of $\text{Fe}(\text{CN})_6^{3-}$ ions in the unperturbed flow and at the wall. In the experiments we simultaneously determined the average heat and mass transfer coefficients over the time and the surface. The heat flux supplied to the experimental section was held constant. Because of the smallness of the concentration of active ions and the variation in the direction of the gradients of concentrations of $\text{Fe}(\text{CN})_6^{3-}$ and $\text{Fe}(\text{CN})_6^{4-}$ ions we can regard the convection that occurred as being thermal.

Figure 2 compares the experimental data on heat transfer with Eq. (1.1) (the solid line). The good agreement of the results is evidence of the clean conditions of the experiment (observance of large volume conditions, absence of secondary circulations of liquid, etc.) and of the correctness of the thermal measurements.

In processing the experimental data on mass transfer it is difficult to choose the temperature at which to determine the thermophysical properties. However, taking into account that in this case $\text{Sc} \gg \text{Pr}$ (the diffusion boundary layer thickness is much less than that of the thermal) one can see that it is most correct to determine these at the wall temperature, and to determine the remaining properties at the average temperature. The parameters in the experiments varied in the following range: $\text{Gr} = (2-100) \cdot 10^6$, $\text{Pr} = 2.5-6$, $\text{Sc} = 200-1000$.

Comparison of the experimental data with the theoretical relations Eqs. (1.11)-(1.13) shows that the experimental mass transfer data are correlated best by Eq. (1.12) (Fig. 3, solid line). As one would expect, Eq. (1.11), which describes mass transfer for $\text{Sc} = 1-15$, somewhat overestimates mass transfer in liquids, while Eq. (1.13), corresponding to infinitely large Schmidt number, underestimates it.

Thus, to calculate mass transfer in liquids under natural thermal convection on a horizontal surface at large Rayleigh number one can recommend Eq. (1.12), which has been confirmed experimentally. From the correspondence between theory and the experimental data in the case of convection in liquids one can postulate that to calculate mass transfer under natural convection in gases one can use Eq. (1.11), which has not been verified experimentally, however.

The results of this work can also be regarded as confirming the correctness of the Leont'ev-Kirdyashkin theory for thermal convection on horizontal surfaces.

LITERATURE CITED

1. S. S. Kutateladze and V. M. Borishanskii, Heat Transfer Handbook [in Russian], Gosenergoizdat, Moscow-Leningrad (1959).

2. A. I. Leont'ev and A. G. Kirdyashkin, "Heat transfer under free convection in horizontal slots in a large volume above a horizontal surface," *Inzh.-fiz. Zh.*, 9, No. 1 (1965).
3. A. G. Kirdyashkin, "Friction and heat transfer under thermal gravitational convection in a centrifugal force field," Author's Abstract of Candidates Dissertation, ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1966).
4. H. Schlichting, *Boundary Layer Theory*, McGraw-Hill (1968).
5. A. P. Burdukov and G. G. Kuvshinov, "Investigation of the boiling mechanism by the electrodiffusion method," in: *Enhancement of Heat Transfer in Power Equipment [in Russian]*, Izd. ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1977).
6. A. P. Burdukov, G. G. Kuvshinov and V. E. Nakoryakov, "Special features of heat transfer in bubble boiling of an underheated liquid in a large volume at subatmospheric pressures," in: *Heat Transfer with Boiling and Condensation [in Russian]*, Izd. ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1978).

OPTIMIZATION OF THE COMPRESSION OF A
SPHERICAL MASS OF GAS

V. I. Kosarev and A. M. Svalov

UDC 533.6.011

One of the basic problems in the study of combustion processes is that of achieving ignition conditions — high enough temperature and density of the combustible medium so that the combustion reaction, once having been started, can effectively continue. To satisfy the ignition conditions it is necessary to impose conditions on the characteristics of the energy source which initiates the reaction. The description of processes taking place during combustion requires taking account of various physical factors such as heat conduction, degeneracy effects, radiation, etc., and the fact that the material being compressed may be a two-component medium — a plasma. Therefore, an analytic study of the problem in the whole volume is practically impossible, and the problem must be separated into parts, each of which can be described by a simpler mathematical model, with a subsequent numerical verification of the limits of admissibility of the simplifying assumptions. Thus, the compression of a material is satisfactorily described by the action of a piston on an ideal compressible fluid

[1-7]. The compression efficiency is commonly characterized by the quantity $\langle \rho R \rangle = \int_0^r \rho dr$,

where ρ is the density of the medium being compressed. It is expedient to divide the solutions under study into two groups; the first group consists of the solutions in which $\langle \rho R \rangle$ reaches the required values when constant input data are used [1-3], and the second group includes all the remaining solutions. It is natural to require simple initial distributions of the hydrodynamic functions so that the proposed schemes can be realized practically by means available. However, taking account of the advance of technical feasibilities and certain advantages of solutions of the second group, it is necessary to study all schemes which ensure the achievement of large values of $\langle \rho R \rangle$. Svalov [7] showed that for special distributions of the initial values solutions exist for which $\langle \rho R \rangle$ becomes infinite at a certain instant for a finite mass and a finite expenditure of energy in the compression, whereas the use of a compression scheme with a uniform deformation [4-6] leads to a linear dependence of $\langle \rho R \rangle$ on the added energy for $\gamma = 5/3$. The solutions given in [7] are self-similar near the origin, and satisfy a system of ordinary differential equations [8]. This system depends on two parameters κ and δ , and the unknown functions can be written in the form

$$u = \frac{r}{t} V(\lambda), \quad \rho = \frac{a}{r^{k+3s}} R(\lambda), \quad p = \frac{a}{r^{k+1s+2}} P(\lambda),$$

$$\lambda = r/(bt^\delta), \quad z(\lambda) = \gamma P(\lambda)/R(\lambda), \quad \kappa = (s+2+\delta(k+1))/\gamma,$$

where u , ρ , and p are respectively the velocity, density, and pressure of a particle, r and t are space and time coordinates, and a , b , k , s , κ , and δ are arbitrary constants.